A STUDY OF TEMPERATURE FLUCTUATIONS IN A PLATE SPECIMEN HEATED WITH ALTERNATING OR PULSATING ELECTRIC CURRENT

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Calculations are shown of temperature fluctuations in a plate specimen heated with alternating or pulsating electric current. The results are compared with measurements of temperature fluctuations in the filament of a heating lamp and in a specimen of zirconium carbide.

For studying the thermal characteristics and the strength characteristics of structural components with internal heat generation, one uses now electrothermal instruments operating on the principle of resistive heating with industrial-frequency alternating current. The resulting thermal flux pulsations can cause appreciable temperature fluctuations in a test structure. This is especially important in the case of refractive materials subjected to extreme temperatures.

Let us calculate the temperature fluctuations in a lengthwise and widthwise isothermal plate containing an internal heat source which varies arbitrarily with time. At small amplitudes ν of the temperature fluctuations ($\nu/T \ll 1$) in a thin plate, we may disregard the temperature-dependence of the thermophysical properties of the plate material. Then the variation of the temperature with time and across the plate thickness will be found by solving the equation of heat conduction

$$c\gamma \frac{\partial T}{\partial t} = \lambda \frac{\partial^2 T}{\partial x^2} + q_v(t). \tag{1}$$

The initial and boundary conditions are

$$t = 0 \quad T = T_0; \tag{2}$$

$$x = 0 \quad \frac{\partial T}{\partial x} = 0; \tag{3}$$

$$x = \frac{a}{2}; \ \frac{\partial T}{\partial x} = -\sigma \frac{\varepsilon}{\lambda} T^4\left(\frac{a}{2}, t\right) - \frac{\alpha}{\lambda}\left[T\left(\frac{a}{2}, t\right) - T_G\right]. \tag{4}$$

Radiative–convective heat transfer is assumed to occur outside the plate. The heat transfer coefficient α and the temperature of the surrounding gas T_G are assumed constant.

Equation (1), with the conditions (2)-(4), was solved by the method of grids [1]. We used the classical implicit scheme on a rectangular grid.

After approximation, we obtained the following system of algebraic equations:

$$\begin{gathered} \frac{\tau}{h^2} T_{n-1}^{m+1} + 2\left(\frac{\tau}{h^2} + \frac{c\gamma}{2\lambda}\right) T_n^{m+1} + \frac{\tau}{h^2} T_{n+1}^{m+1} = -\frac{q_v(m\tau)}{\lambda} - \frac{c\gamma}{\lambda} T_n^m, \\ T_1 = T_0, \ m = 0, 1, \ldots, \infty; \quad n = 1, 2, \ldots, N-1, \\ T_N = \left(T_{N-1} - \frac{h\varepsilon\sigma}{\lambda} T_N^4 + \frac{\alpha h}{\lambda} T_G\right) \frac{1}{1 + \frac{\alpha h}{\lambda}}. \end{gathered} \tag{5}$$

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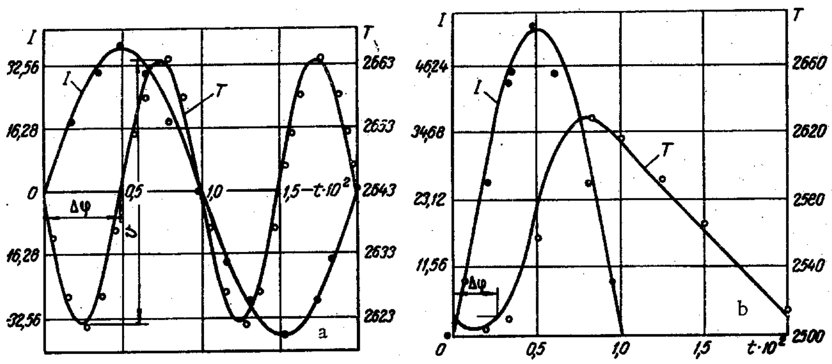

Fig. 1. Fluctuations of the current (I, A) and the temperature (T, °K) of a tungsten filament, with time (t, sec) in a lamp energized with (a) alternating current or (b) pulsating current; curves represent calculated values. T = 2643°K.

System (5) was solved by the elimination method [2]:

$$T_n = L_n T_{n+1} + K_n,$$

where coefficients L_n, K_n had been defined in [2] and T_N was found from the solution to the following system of algebraic equations:

$$T_{N-1} = L_{N-1} T_N + K_{N-1},$$
$$T_N = \left(T_{N-1} - \frac{h\varepsilon\sigma}{\lambda} T_N^4 + \frac{\alpha h}{\lambda} T_G\right) \frac{1}{1+\frac{\alpha h}{\lambda}}. \tag{6}$$

System (6) yielded a fourth-degree algebraic equation

$$\frac{h\varepsilon\sigma}{\lambda} T_N^4 + \left(1+\frac{\alpha h}{\lambda} - L_{N-1}\right) T_N - K_{N-1} - \frac{\alpha h}{\lambda} T_G = 0,$$

which was then solved by the method of secants.

In order to verify this method of computing the temperature fluctuations, we have performed experiments in measuring the temperature fluctuations in a model SI-10-300 heating lamp. Not only was such an experiment simple, but the tungsten filament could also be reliably regarded as a plate specimen (20 × 2.77 × 0.04 mm) with known and stable thermophysical properties.

The heating lamp was energized from an ac network through a step-down transformer. The filament temperature was regulated by means of a model RNO-250-2 autotransformer. The lamp current was measured with a model D553 ammeter of class 0.2. The fluctuations in the lamp brightness were measured with a special-purpose photoelectric converter and recorded on the screen of a model S1-17 oscillograph. A signal proportional to the filament current, i.e., variations of the filament current and of the lamp brightness with time, were also recorded on the same screen so as not to distort their phase relation. The brightness fluctuations were then converted into temperature fluctuations according to the formula

$$\Delta T = \frac{\lambda_1 T^2}{C_2} \cdot \frac{\Delta B}{B}$$

on the basis of Wien's law. The mean filament temperature was measured with a model ÉOP-51 optical pyrometer. Brightness temperature was then converted into true temperature with the aid of the relation derived on the basis of the variability of monochromatic emissivity with temperature and taking into account the absorption of radiant energy in the sighting window of the lamp.

The temperature fluctuations and the phase shift were measured over the 1600-2400°K range of brightness temperatures.

The test results are shown in Fig. 1a, b. The effects of a rising mean filament temperature on the amplitude of temperature fluctuations and on the phase shift are shown in Fig. 2, along with calculated temperature fluctuations.

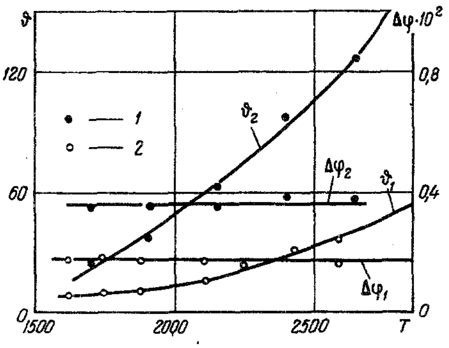

Fig. 2

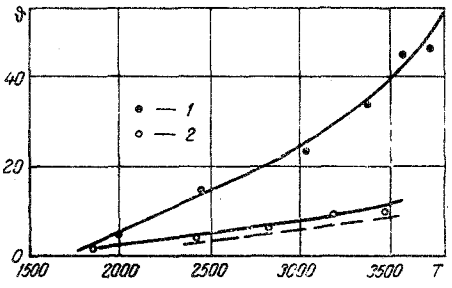

Fig. 3

Fig. 2. Amplitude of temperature fluctuations ϑ (°K) and phase shift $\Delta\varphi$ (sec) as functions of the mean filament temperature T (°K) in a lamp energized with (1) alternating current or (2) pulsating current.

Fig. 3. Amplitude of temperature fluctuations (ϑ, °K) in a plate specimen of zirconium carbide, as a function of the mean temperature (T, °K) when heated with (1) alternating current or (2) pulsating current; dashed line represents calculations for alternating current.

Since the filament enclosure was 60 mm in diameter, it was possible to roughly estimate the convective dissipation of heat from the filament by assuming here natural convection in a volume. According to such an estimate made by a well-known method [3], at filament temperatures above 1600°K the convective heat transfer amounts to less than 10% of the radiative heat transfer. As the temperature rises, the contribution of convection to the overall heat balance decreases fast and may be disregarded in any calculation of temperature fluctuations.

When a lamp is energized with current at industrial frequency, then the internal heat generation varies with time according to

$$q_v(t) = \frac{1}{F^2} I^2(t)\,\rho. \tag{7}$$

The instantaneous current is

$$I(t) = I_{max} \sin\left(\frac{2\pi}{\tau_n} t\right), \tag{8}$$

and the rms current

$$I_{rms} = \frac{1}{\sqrt{2}} I_{max} \tag{9}$$

determines the mean filament temperature, which, to the first approximation, is taken as the initial temperature T_0:

$$T_0 = \frac{I_{rms}^2 \rho}{\varepsilon\sigma ab(a+b)}. \tag{10}$$

Since ε and ρ are known functions of the temperature, hence T_0 can be calculated according to Eq. (10) by the iteration method for any given value of I_{rms}.

Thus, when the value of I_{rms} is given, expressions (7)-(9) yield the law according to which the heat generation varies with time, while the initial value T_0 is determined from expression (10). In our calculations of temperature fluctuations, the values of all thermophysical properties of the material are referred to T_0.

It is to be noted that the results of calculations will be as accurate as the value given for the initial temperature. Thus, if the initial temperature is specified with an error within ±100°C, then the regular heating mode will be reached within 15-20 iteration cycles.

This numerical method of calculation was used for determining the variation of temperature with time as well as the amplitude of temperature fluctuations ϑ. From the known variation of current with time and the resulting temperature variation, we then determined the phase shift $\Delta\varphi$ between current and temperature. The calculations were made on an M-20 digital computer. The deviation of test points from calculated values was not in excess of 7-8%.

When the lamp was energized with pulsating current, the amplitude of temperature fluctuations increased by a factor of 3 and the phase shift increased by a factor of 2. For the same mean temperature, the maximum value of pulsating current was 20% higher than that of alternating current.

In Fig. 3 are shown test data on the temperature fluctuation in a plate specimen ($100 \times 2.2 \times 0.7$ mm) of zirconium carbide heated with industrial-frequency alternating current. With a change from alternating to pulsating current, the amplitude of fluctuations increased by a factor of approximately 3.5 and reached the $\pm 0.9\%$ level at T = 3600°K.

NOTATION

x	is the plate-thickness coordinate;
t	is the time;
c	is the specific heat;
γ	is the density;
λ	is the thermal conductivity;
ρ	is the electrical resistivity;
q_V	is the internal heat generation;
T	is the temperature;
ε	is the integral emissivity;
a	is the thickness;
b	is the width;
F	is the cross-section area;
τ	is the time interval in computation;
h	is the thickness step in computation;
σ	is the Stefan–Boltzmann constant;
$1/\tau_n$	is the frequency of current fluctuations;
B	is the lamp brightness;
λ_1	is the effective wavelength at which brightness is measured;
$C_2 = 1.439$ cm · °K	is the constant in Wien's law.

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